

# Anomalous Coherence of Longitudinal Oscillation of Beam Bunches in a Synchrotron

J. A. MacLachlan

*Fermi National Accelerator Laboratory\**

Box 500, Batavia IL 60510

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## Abstract

It has been observed that the coherent longitudinal oscillation of bunches injected off the synchronous trajectory in the Fermilab Main Injector persists for hundreds of milliseconds whereas the expected coherence time is about 10 ms. This circumstance is fortunate with respect to the power requirement for a broadband longitudinal damper system, but the phenomenon has not been well understood, and it is not obvious over what parameter range it can be expected. An investigation by macroparticle modeling has identified relevant beam and machine parameters and the underlying mechanism.

## 1 Introduction

The broadband longitudinal damper for the Fermilab Main Injector (FMI)[1] has been developed on the basis of an observation that the injection phase error signal persists for much longer than one would expect from single particle dynamics. The natural assumption is that a bunch injected out of synchronism will smear out or filament to fill a larger effective longitudinal emittance within a few synchrotron oscillation periods. Figure 1 illustrates the phase space distribution of a 0.07 eVs bunch in the FMI modeled with the single particle equations of motion 20 ms, or 17 synchrotron oscillation

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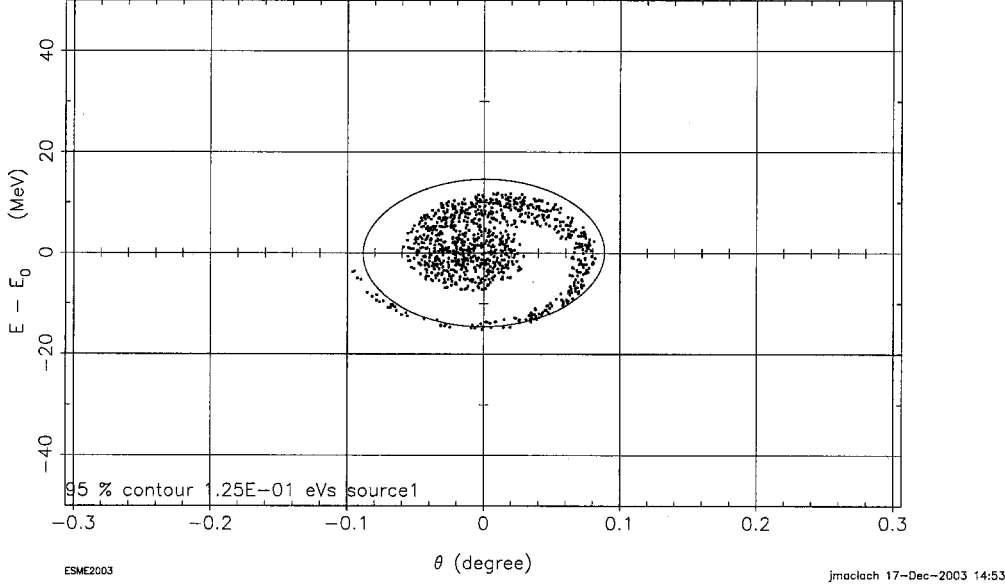


Figure 1: The phase space distribution of a 0.07 eVs bunch in the FMI 20 ms after injection 20 degrees off phase as modeled with single particle equations of motion. The effective emittance at this time is about 0.11 eVs, but it will grow further without dipole damping.

periods, after injection twenty degrees (1.05 ns) off the synchronous phase. Figure 2 shows the centroid phase error over this time interval. Clearly there is little useful error signal after about 15 ms, so in the FMI there must be some collective force that acts to keep the bunch together. The source of all collective forces on the bunch is ultimately the electric charge of individual protons in the bunch. Therefore, whatever the mechanism, one must expect that bunch intensity is a crucial parameter. Though intuitively one expects the space charge force to disrupt the bunch because the interparticle force is repulsive, there is at least some regime below transition energy in which the collective voltage is focusing, perhaps a counter-intuitive observation, and, to the author's knowledge, a novel one.

## 2 Charge Dependent Bunch Coherence

Figure 3 is the result of tracking the evolution of a 0.07 eVs bunch of  $6 \cdot 10^{10}$  protons injected twenty degrees off phase. The *only* source of longitudinal

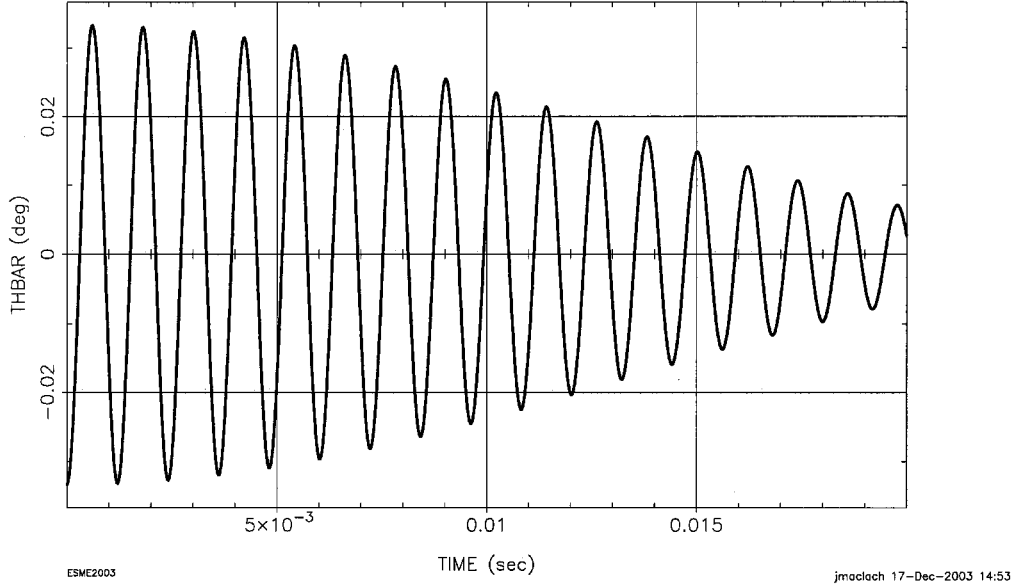


Figure 2: The phase error signal produced by the bunch in Fig. 1 during the 20 ms of filamentation of the distribution.

coupling impedance is the perfectly conducting wall space charge:

$$\frac{Z_{\parallel \text{spchg}}}{n} = -i \frac{Z_o g}{2\beta\gamma^2} , \quad (1)$$

where  $Z_o$  is the free space impedance of 376.7 Ohms,  $g \approx 5$  is the so-called geometric factor, and  $\beta$  and  $\gamma$  are the relativistic velocity and energy parameters. The qualitative information which can be extracted from this result is that the major fraction of the charge is concentrated in a clump not very different in area from the original bunch, and there is some smearing of charge along the phase space trajectory which is the expected boundary of the filamentation. The phase error signal over the 20 ms which resulted in the distribution of Fig. 3 is plotted in Fig. 4, which should be compared to the  $q=0$  result in Fig. 2. Similar modeling shows the enhanced coherence to be useful above an intensity of  $3 \cdot 10^{10}$  per bunch up to several  $\times 10^{11}$ , becoming stronger as the charge is increased.

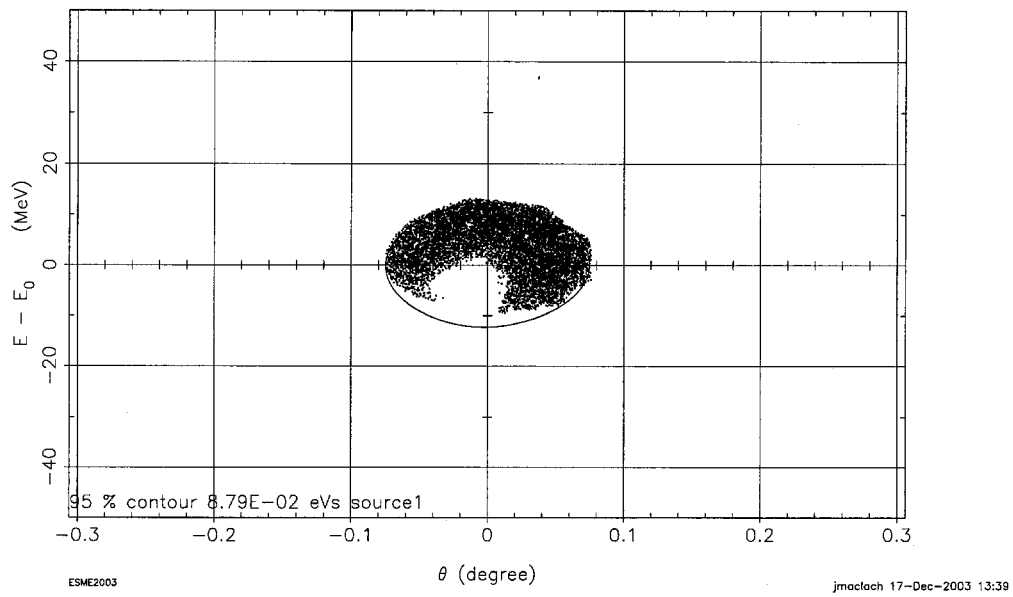


Figure 3: The phase space distribution of a 0.07 eVs bunch of  $6 \cdot 10^{10}$ /bunch in the FMI 20 ms after injection 20 degrees off phase. The effective emittance is about 0.08 eVs, which will slowly increase during further oscillation without dipole damping.

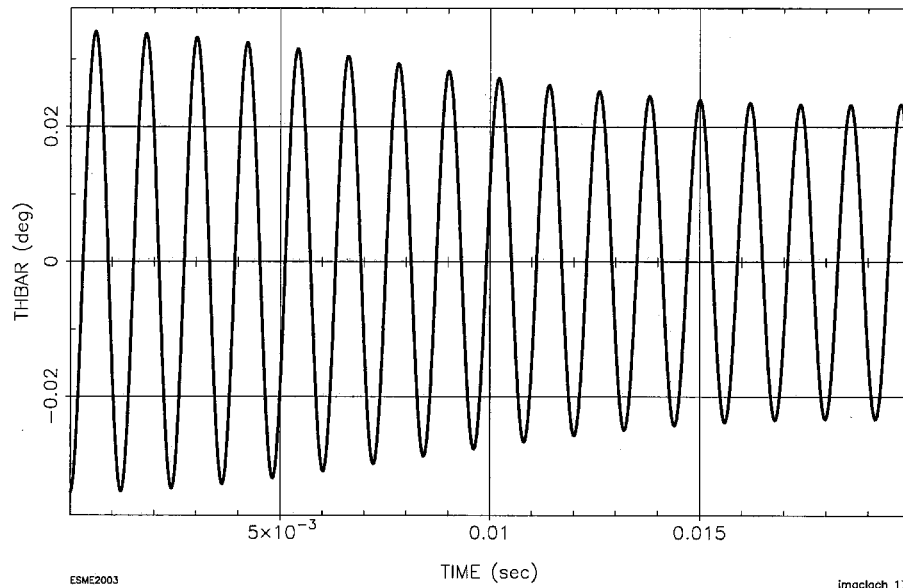


Figure 4: The phase error signal produced by the bunch in Fig. 3 during the 20 ms of its development from a matched bunch injected 20 degrees off the synchronous phase.

### 3 Beam Physics

The mechanism underlying the puzzling persistence of a coherent bunch oscillation is at once simple and paradoxical. It is generally understood that the perfectly conducting wall space charge force is repulsive and defocuses the incoherent oscillation of particles within the bunch; however, the effect of the potential distortion is to produce high *focusing* gradients at the bunch ends, creating a self-induced isolated bucket. A form for the perfectly conducting wall potential entirely equivalent to that in eq. 1 is

$$E_{sc} = -\frac{g}{4\pi\epsilon_0\gamma^2} \frac{\partial\lambda}{\partial z} \quad , \quad (2)$$

where the potential is just the constant field  $E_{sc} \times$  the ring circumference and  $\lambda$  is the linear charge density of the bunch. For a typical smoothly distributed bunch, the derivative of  $\lambda$  gives a leading positive peak and a similar trailing negative one. The perfectly conducting wall potential is plotted for a bunch of  $2 \cdot 10^{11}$  and parameters characteristic of the FMI in Fig. 5. In Fig. 6 is plotted a phase space distribution of 0.07 eVs containing  $6.4 \cdot 10^5$  macroparticles representing  $2 \cdot 10^{11}$  protons in the FMI. The dotted curve in this figure is the sum of the rf potential and the space charge potential shown in Fig. 5. The steep local gradients at the bunch ends are apparent by inspection. The potential distortion occurs wherever the bunch is located; it moves with the bunch oscillation and thus continuously maintains the bunching. The condition for the stable collective motion is basically set by the brightness of the bunch because the potential is stronger for a bright bunch and the range of momenta to be contained is lower for low emittance. At the intensity threshold for enhanced coherence, the centroid oscillation amplitude begins to decline in a manner rather similar to the charge zero case but then builds up and maintains over at least 150 ms a value not much less than its initial one. Comparison of the charge zero and  $q=3 \cdot 10^{10}$  protons/bunch amplitudes of phase oscillation in Figs. 7 and 8 illustrates this point. That is, the evidence favors the interpretation of a stable or nearly stable charge configuration forming above a certain intensity threshold. The self-induced bucket interpretation is that the induced local bucket becomes large enough at threshold intensity to contain the bunch.

The case treated is for injection into the FMI, which takes place below the transition energy. Because the sign of focusing for the incoherent motion changes at transition, it is natural to question whether the effect on the

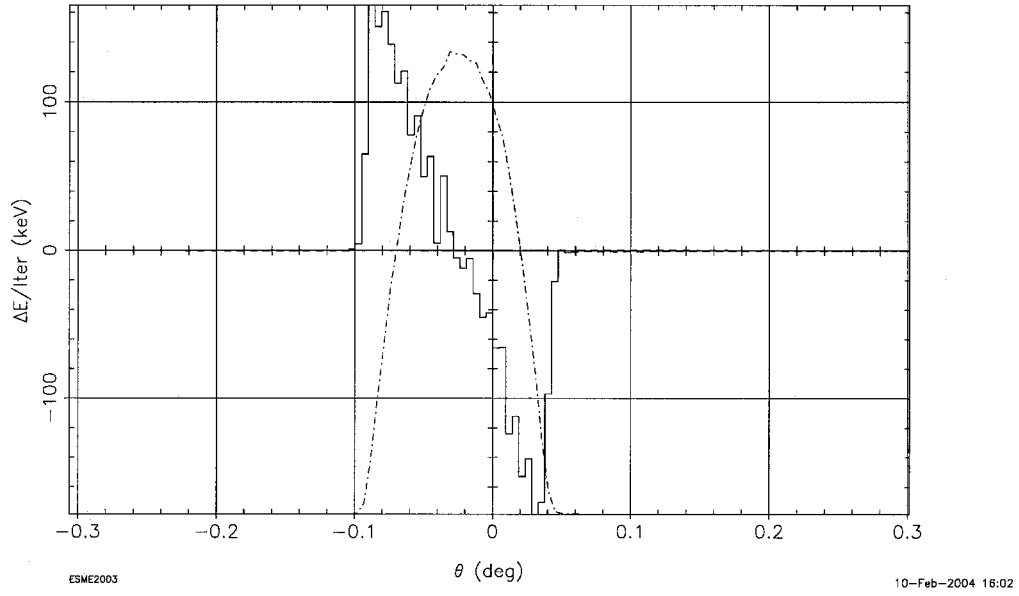


Figure 5: The perfectly conducting wall space charge potential for a 0.07 eVs bunch of  $2 \cdot 10^{11}$  protons using FMI parameters. The dashed curve is the linear charge density, plotted to arbitrary scale.

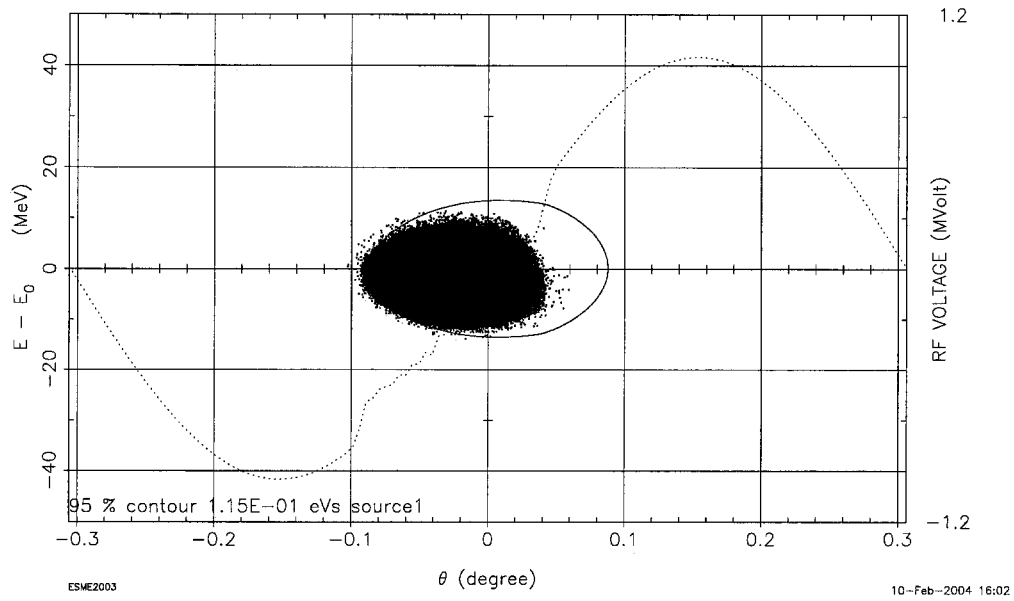


Figure 6: Phase space distribution of a hypothetical FMI bunch of 0.07 eVs containing  $2 \cdot 10^{11}$  protons 44 ms after injection  $20^\circ$  off synchronous phase. The dotted curve is the sum of the 1 MV peak rf potential and the space charge potential plotted in Fig. 5.

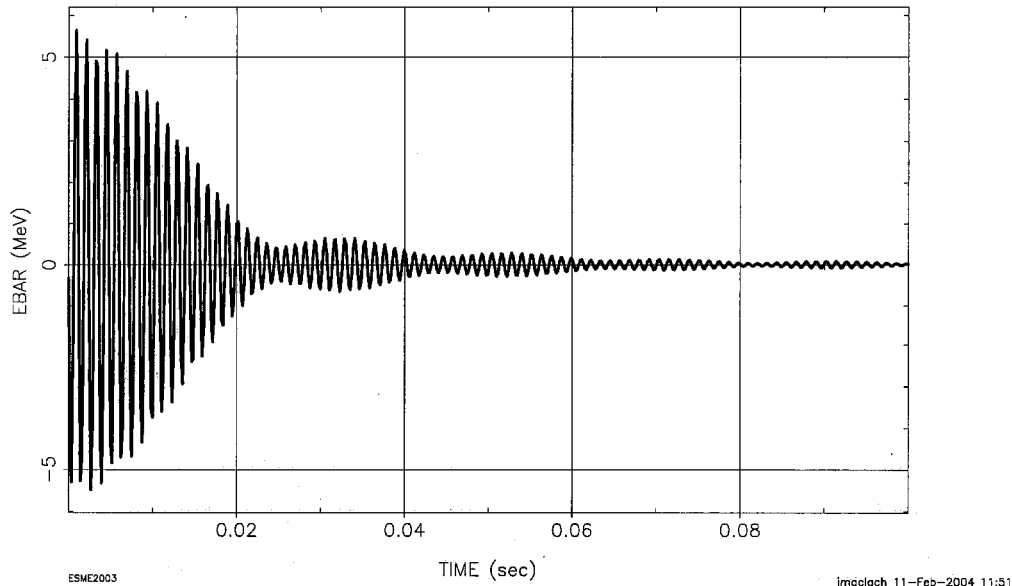


Figure 7: The amplitude of the phase error of the bunch centroid for a 0.07 eVs bunch in the FMI with no charge, started  $20^\circ$  off synchronous phase.

bunch containment will likewise change sign at transition. This is a somewhat academic question because there is usually little need for damping of large phase errors at higher energies and because the space charge force is much reduced by  $\gamma^{-2}$  in the coefficient of the charge gradient in eq. 2. However, by using a fictional  $\gamma_T = 7.14$  as a FMI parameter, the slip factor  $\eta = \gamma_T^{-2} - \gamma^{-2}$  has the original magnitude but positive sign. Fig. 9, having otherwise all of the conditions in Fig. 6, shows that indeed the confinement of the bunch is lost. It is interesting to see the coherence of that part of the bunch which has not encountered the defocusing potential at the back and front of the distribution; one sees here the *focusing* effect of the space charge force on the incoherent (individual particle) motion above transition, manifest in the increased gradient of the total potential over the phase range spanned by the bunch evident in Fig. 9.

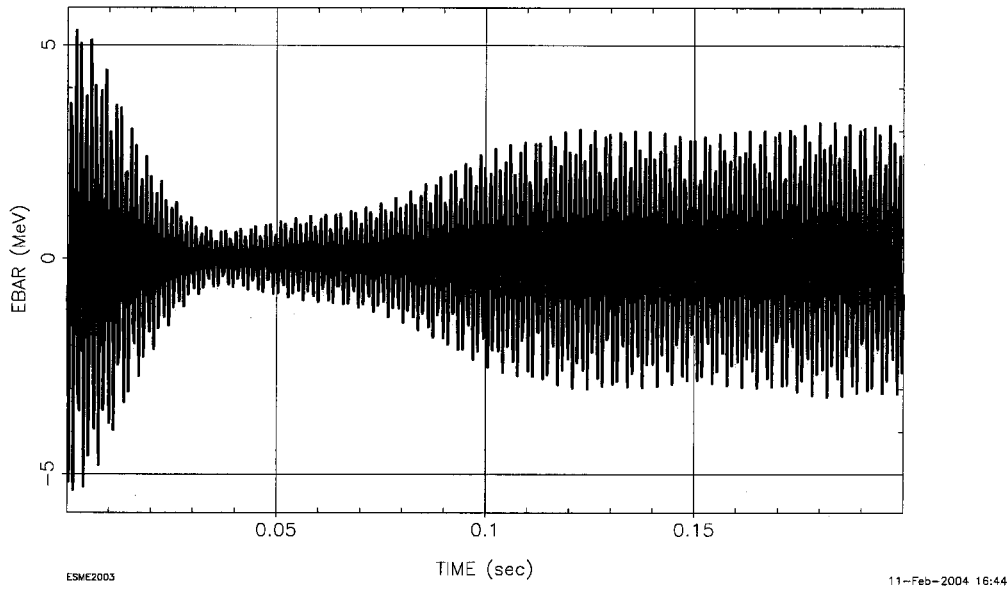


Figure 8: Like Fig. 7, the amplitude of phase error for the bunch centroid for a 0.07 eVs bunch started  $20^\circ$  off synchronous phase. In this case, the charge of  $3 \cdot 10^{10}$  protons is included.

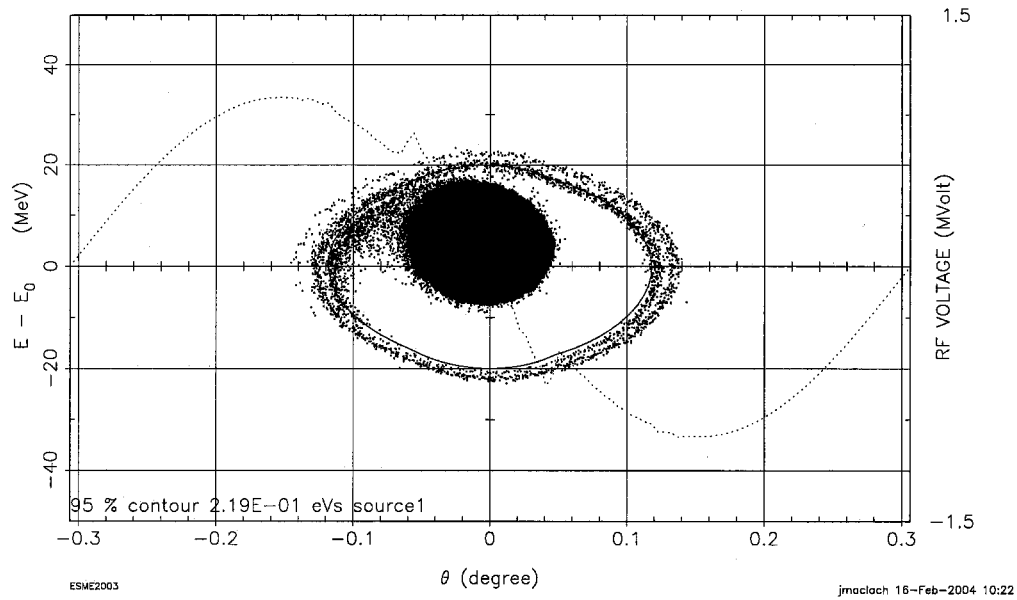


Figure 9: Like Fig. 6, the phase space distribution of a hypothetical FMI bunch of 0.07 eVs containing  $2 \cdot 10^{11}$  protons 44 ms after injection  $20^\circ$  off synchronous phase. The dotted curve is the sum of the 1 MV peak rf potential and the space charge potential plotted in Fig. 5. However, the sign of the slip factor  $\eta$  has been changed from negative to positive; *i. e.*, the motion takes place above transition.

## 4 Modeling of Damping

A practical question is whether the enhanced coherence will facilitate damping with a low power broadband damper. The statistical emittance *vs.* time is shown over 0.5 s without damping in Fig. 10 and with 2 kV damping in Fig. 11. The 95 % emittances of the bunches are not very different at 0.5 s — 0.112 eVs with no damping, and 0.110 with. The bunch brightness, however, as reflected in the statistical emittance, is substantially higher after feedback. The damping is not complete even after a half second with the damper strength used, so there is still centroid offset affecting the effective emittance. The phase error signal remains strong over the full time, and it is possible to further damp the motion of the persistent clump with 2 kV from broadband damper cavities. Ideally one would like faster damping, but much of the original beam brightness has been preserved even though the boundary for the full distribution is similar to that obtained for no charge and no feedback shown in Fig. 1. It may be possible to get somewhat higher damper voltage; also, however, injection errors may be routinely controllable to better than twenty degrees. If the voltage on the damper is 8 kV, the 20 degree injection error is damped in 0.2 s or so; the plot of statistical emittance *vs.* time is shown in Fig. 12. Precise evaluation of damping time and final emittance can be expected to depend at least weakly on other sources of coupling impedance. Two different estimates of  $Z_{\parallel}$  were tried individually in combination with  $Z_{\parallel \text{spchg}}$  without qualitative change in the results. Because the actual longitudinal coupling impedance is probably comparable to one or the other of the estimates used, it seems appropriate to ignore it in the present context.

## 5 Summary

The apparently anomalous persistence of coherent longitudinal oscillation for bright bunches injected into the FMI with centroids off the synchronous energy and phase values has been explained as a self-induced focusing arising from the perfectly conducting wall space charge potential, a previously undescribed collective effect. Modeling using FMI parameters has shown that the phenomenon reduces the voltage required for broadband dampers at injection, a practical benefit of the often detrimental space charge force.

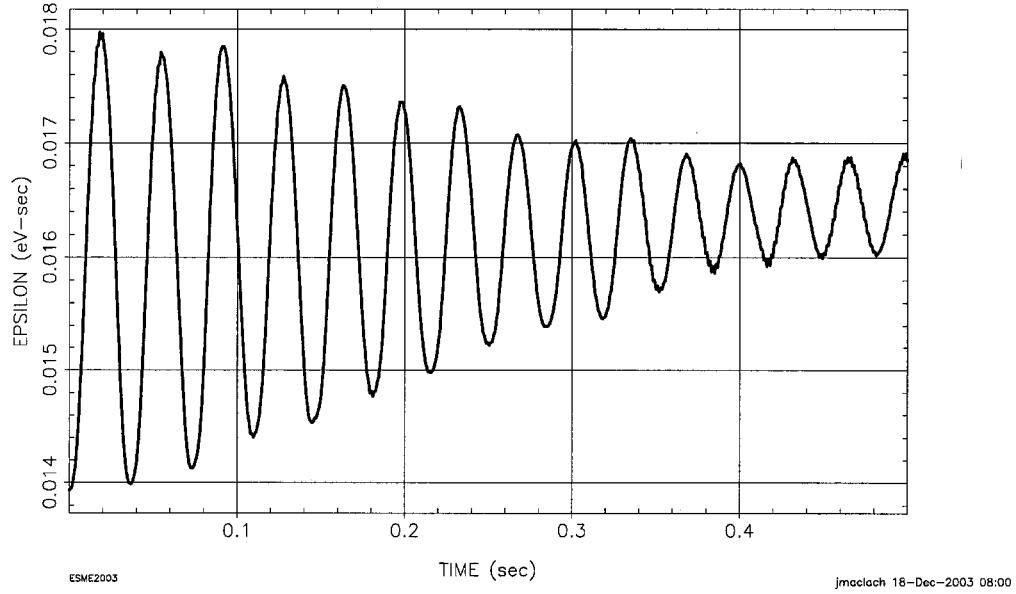


Figure 10: Statistical (rms) emittance of a 0.07 eVs bunch of  $6 \cdot 10^{10}$ /bunch in the FMI for 0.5 s after injection 20 degrees off phase with no phase feedback.

## References

- [1] G. William Foster *et al.*, “BUNCH-BY-BUNCH DIGITAL DAMPERS FOR THE FERMILAB MAIN INJECTOR AND RECYCLER”, 2003 US PAC, Portland OR (2003) <http://accelconf.web.cern.ch/accelconf/p03/PAPERS/TOPD003.PDF>

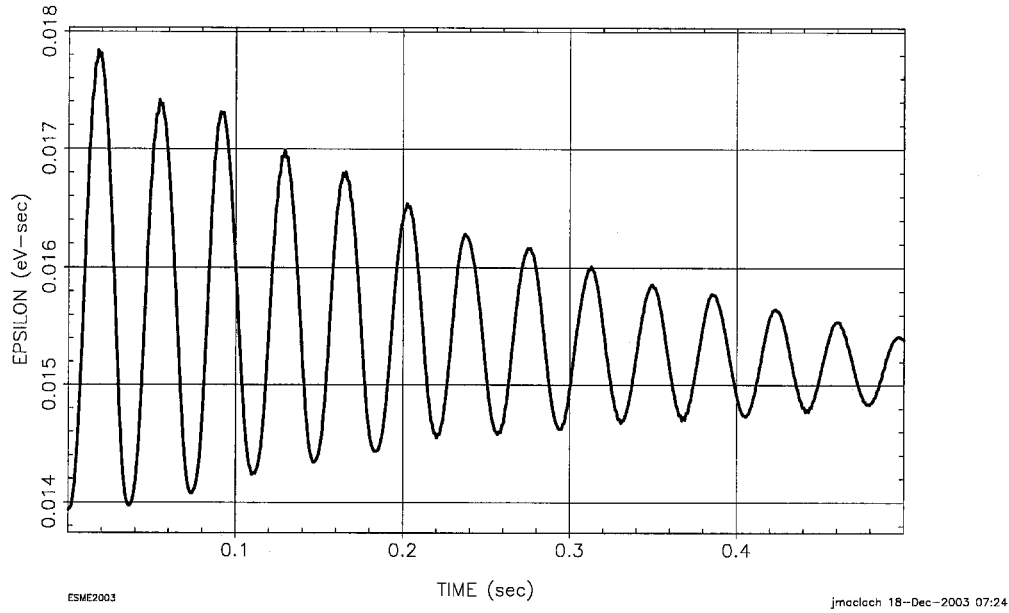


Figure 11: Statistical (rms) emittance of a 0.07 eVs bunch of  $6 \cdot 10^{10}$ /bunch in the FMI for 0.5 s after injection 20 degrees off phase with phase feedback provided by a 2 kV broadband damper.

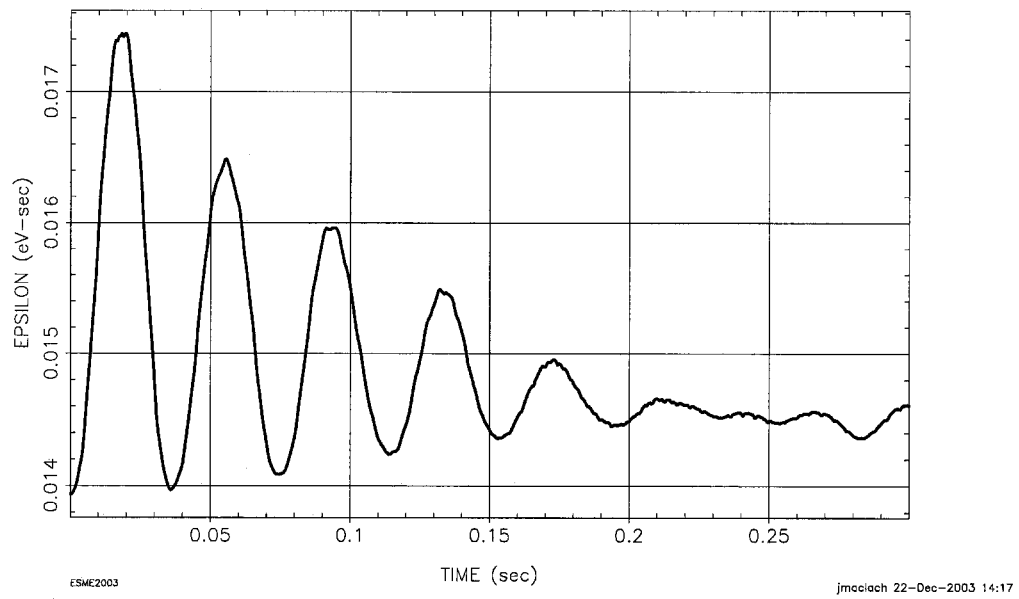


Figure 12: Statistical (rms) emittance of a 0.07 eVs bunch of  $6 \cdot 10^{10}$ /bunch in the FMI for 0.3 s after injection 20 degrees off phase with phase feedback provided by a 8 kV broadband damper.

